

Introduction

This booklet is intended to clarify and explain some of the ways in which your child is taught to write down calculations. You may be surprised to find your child's Mathematics book contains writing, pictures, diagrams, jottings or blank number lines and not many 'formal calculations'. Certainly, many of the methods your child uses may be very different from those that you learned at school. From the very early years at school the emphasis on mathematics learning is upon children understanding and applying the skills they have learnt rather than just learning a formal written method that they may not fully understand.

Younger children, in KS1, record their ideas in a variety of ways that do not necessarily look like the kind of 'sums' you might remember. This is because written calculations are not the ultimate aim. Our aim is for the children to look at a calculation and to first ascertain if they can do it in their heads, estimating first and then, if the numbers are too large, to use a way of writing them down that aids their own thinking. You may also find that the methods you learnt at school are taught later on in your child's schooling because of greater emphasis on mental calculations. Here in this booklet I have tried as simply as possible to aid you in understanding some of the strategies your children may use, particularly at KS2, to help you, to help your children. You'll be amazed at how many different ways there are to tackle the same concept! The booklet contains the approaches your children may use to support their thinking when solving addition, subtraction, multiplication and division calculations. I have tried to make the strategies as clear as possible; however, if you are unsure of any ideas in the booklet please do not hesitate to ask your child's class teacher or myself.

Mrs A Painter
Mathematics Co-ordinator



Looking at addition & subtraction

When children are in Foundation Stage and Year One they are **not** expected to do vertical sums like
$$\begin{array}{r} 6 \\ +4 \\ \hline 10 \end{array}$$
 but that doesn't mean they won't learn that $6+4=10$.

They will be doing a daily mixture of practical, mental and oral work including lots of counting, talking about numbers and using numbers in real life activities. They will begin to record what they've done with pictures and numbers. These recordings will help them to understand what is happening and to show how they've worked something out. Here are two examples of early recording.

Jane had 3 bears. She was given 2 more. How many does she have now?

3 $3 + 2 = 5$ 2

There were 8 balloons. Two popped. How many are left?

$8 - 2 = 6$

This next example shows how different children have worked out and recorded the answer to the same problem about the children in the class.

There are 20 children in our class. Three are away today. How many are here?

17 here

2 away would be 18
so 3 away must be 17.

$20 - 3 = 17$

These diagrams and jottings help the children to see what is happening to the numbers and to use some facts they already know to help them work out others.

After this children will carry on using horizontal recording of addition and subtraction to support their mental calculations. The example below shows two ways of adding 76 and 93. The first splits the numbers into tens and ones (units) then adds the tens followed by the ones to give 169. The second example uses the idea of rounding to 100 by taking 7 from 76 and adding it to the 93 to make it 100 so making the addition easier.

In a school there are 76 boys and 93 girls. How many children are there altogether?

$$\begin{aligned} 93 + 76 &= 90 + 70 + 3 + 6 \\ &= 90 + 70 + 9 \\ &= 160 + 9 \\ &= 169 \end{aligned}$$

$$\begin{aligned} 93 + 76 &= 93 + 7 + 76 - 7 \\ &= 100 + 69 \\ &= 169 \end{aligned}$$

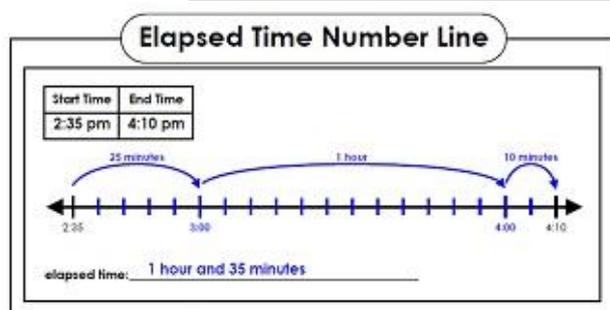
Children will also continue to use drawings, diagrams and blank number lines to support their thinking, as below.

There are 13 boys and 8 girls in the room. How many altogether?

$13 + 8$

There are 34 children in the classroom. 27 go to the hall. How many are left?

$34 - 27$:



The next stage is that children will be taught written methods, including vertical addition and subtraction, using an 'expanded method', and also subtraction using number lines by counting on (complementary addition). This form of recording will be used for those calculations that they can't do 'in their heads'. 'Expanded methods' are ways of recording that make the process of adding the different digits clear to children. These methods build on the mental methods they have been learning and should help children to understand what is happening. Here is an example of adding using an expanded method.

The blue team's score of 287 points is increased by 145 points. What's the new score?

287	→	200	+	80	+	7
+ 145	→	100	+	40	+	5
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432	←	300	+	120	+	12

This is the same question set out in a different way but using the same ideas. It includes an explanation of what might be said to describe the adding.

The blue team's score of 287 points is increased by 145 points. What is the new score?

$$\begin{array}{r}
 287 \\
 + 145 \\
 \hline
 120 \\
 120 \\
 \hline
 300 \\
 120 \\
 \hline
 432
 \end{array}$$

Explanation.....

The language used is very important to help children understand the size of numbers being added (e.g. is it seven or seventy or seven hundred)

seven plus five equals twelve

eighty plus forty equals one hundred and twenty

two hundred plus one hundred equals three hundred

finally **three hundred plus one hundred and twenty plus twelve equals four hundred and thirty two**

Expanded version of subtraction:

$$\begin{array}{r}
 89 = 80 \quad 9 \\
 -57 \quad 50 \quad 7
 \end{array}$$

$$30 + 2 = 32$$

Moving on to adjusting if appropriate:

$$\begin{array}{r}
 71 \\
 -46 =
 \end{array}$$

Step 1 $70 + 1$
 $-40 + 6$

Step 2 $60 + 11$
 $-40 + 6$

$$20 + 5 = 25$$

Which is recorded by children as:

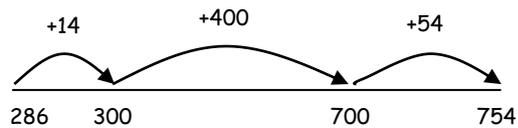
$$\begin{array}{r}
 {}^{60}70 + {}^{11}1 \\
 - 40 + 6
 \end{array}$$

$$20 + 5 = 25$$

These methods mean that children may have to write a little more at this stage but, because it helps and supports their understanding, it enables them to become much more confident and quicker in the long run.

Here is an example of a subtraction problem involving 3 digit numbers:

$$754 - 286 = 468$$



OR

$$754 - 286 = 468$$

14 (300)	can be refined to	14 (300)
400 (700)		<u>454</u> (754)
<u>54</u> (754)		468
468		

Here you can see that children are using an image of a number line to support their thinking to work out this subtraction. Children are encouraged to find the difference between 286 and 754 by counting on, just as shopkeepers work out change, and how *you* might calculate mentally. Beginning at 286, 14 is needed to count on to the next 'friendly' number, 300; then a further 400 to land on 700; and finally another 54. All of the jumps total 468. Without the number line the calculation might look like the second example. This method of subtraction is known as 'complementary addition'.

Try to decide how you would do the following calculations. Would you do them in your head, write them down or use a calculator? The notes may change your mind but don't read them until you've had a go at the calculation.

45+99	3006-2999	2.3 + 6.99	4532-3768	The total of all the numbers from 1 to 10
This is easy to do if you think of 99 being one less than 100. So add 100 to make 145 and then take away 1 to give 144.	These numbers are very close on the number line. We need 1 to get from 2999 to 3000 and 6 more to get to 3006 so the difference is 7. So much easier than doing a vertical sum!	This addition involving decimals may look hard but it's easier if you think of it as money. £2.30 add £6.99. So add £7 to get £9.30 and take away 1p to get £9.29 Easy!	This, for most people, needs a pencil and paper.	This is easy if you think of pairs of numbers making 10 $1 + 9 = 10$ $2 + 8 = 10$ $3 + 7 = 10$ $4 + 6 = 10$ Finally $5 + 10 = 15$ giving a total of 55

It is important that calculations are presented in problems or horizontally, as above, to encourage children to think about the numbers as a whole, what they mean, what a sensible answer might be and the best method of working them out. It's all about giving children confidence with and control over numbers.

Following this pupils will move onto the formal written method for addition and subtraction that you may have learnt at school. Here are some examples:

$$\begin{array}{r}
 13587 \\
 + 1675 \\
 \hline
 15262 \\
 \hline
 111
 \end{array}$$

$$\begin{array}{r}
 3 \quad 6 \quad 7 \quad 11\cancel{1} \\
 - 1 \quad 4 \quad 6 \\
 \hline
 2 \quad 2 \quad 5
 \end{array}$$

Looking at multiplication & division

Did anyone ever tell you that you only needed to learn about half of the multiplication tables in order to know them all? If they didn't it was a bit mean because if you know $3 \times 4 = 12$ you also know $4 \times 3 = 12$, so why learn it twice?

Did anyone ever say that once you knew the 2 times table, which is only double the 1 times, then the 4 times was easy because you just double the 2 times? Then you can double the 4 times to get the 8 times. The 3 times doubles to the 6 times and 12 times, the ten times can be halved to give 5 times and so on. This uses the knowledge children are developing through addition and subtraction and makes important connections for them. This chart shows how this works for the 2x, 4x and 8x tables.

x	1	2	3	4	5	6	7	8	9	10
2times	2	4	6	8	10	12	14	16	18	20
4times	4	8	12	16	20	24	28	32	36	40
8times	8	16	24	32	40	48	56	64	72	80

double
↓

It's also possible that you weren't told that you knew your division tables. If you were shown that division was the opposite of multiplication you will understand that knowing $3 \times 4 = 12$ or $4 \times 3 = 12$ also means you know $12 \div 4 = 3$ and $12 \div 3 = 4$. So knowing one number fact, like $3 \times 4 = 12$, immediately means we know at least four.

But did you also realise that knowing any one of these facts helps you to know a lot more than four without actually learning them? Read on.

The early work children do will introduce them to the ideas of multiplication and division. They will be counting in different patterns, helped to see how multiplication is repeated addition and division is repeated subtraction, shown how division is the opposite of multiplication and taught to understand place value (that in 234 the 2 is 200, the 3 is 30 and the 4 is 4 ones (units)). This knowledge and understanding, with much of the work being done in their heads, opens up a whole world of facts for them and they don't all have to be memorised. That can make dealing with numbers feel a lot easier.

The following chart shows something of what this means.

If you know $4 \times 5 = 20$ what else do you know?

$$5 \times 4 = 20$$

$$20 \div 4 = 5$$

$$20 \div 5 = 4$$

$$4 + 4 + 4 + 4 = 20$$

$$5 + 5 + 5 + 5 = 20$$

and using knowledge of place value

$$5 \times 40 = 200$$

$$4 \times 50 = 200$$

$$200 \div 50 = 4$$

$$200 \div 40 = 5$$

$$40 \times 50 = 2000 \text{ and so on}$$

combined with knowing the multiples of ten, which are easy, we can work out that

$$5 \times 44 = 220 \quad (5 \times 40 = 200 \text{ and } 5 \times 4 = 20 \dots 200 + 20 = 220)$$

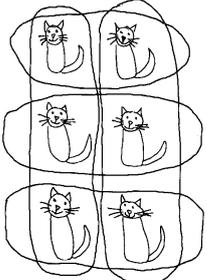
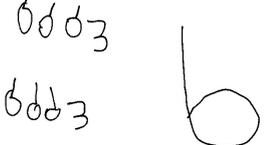
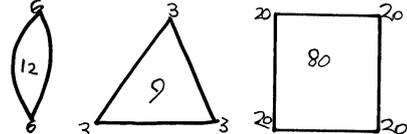
therefore

$$220 \div 5 = 44$$

and so it goes on. Try it for yourself..in your head!

The ability to do what you've just seen, developed gradually through years 1,2,3 & 4 helps children in years 4,5 & 6 to move on with confidence to multiplication and division of bigger numbers including those involving decimals and as with addition and subtraction, the questions will usually be presented to children as word problems or, horizontally, as calculations. The children should then be encouraged to work them out mentally if they can (supported by drawings, diagrams, number jottings if necessary) or, if they can't, to use the most suitable written method they know.

To begin with the children will be recording to demonstrate how they have done something and to show that they've understood what is happening, as below.

		
<p>2×3 cats = 6 cats or 3×2 cats = 6 cats</p>	<p>2 lots of 3 apples makes 6 apples.</p>	<p>$12 = 2 \times 6$ $9 = 3 \times 3$ $80 = 4 \times 20$</p> <p>$80 \div 4 = 20$ $80 \div 20 = 4$</p>

In years 3 & 4 the children will begin to use expanded methods to help them deal with calculations that they can't do in their heads. At this stage it will mostly involve multiplying and dividing 2 digit numbers by a single digit (72×6 or $85 \div 4$). When dividing they will learn about and use remainders.

The expanded method for multiplying is often called the grid method. It uses the mental skills and the knowledge children have been learning and will help most children to move, with understanding, to the 'compact' method you may know.

This chart shows 'the grid method'. You can see, as with addition and subtraction expanded methods, it uses knowledge of number facts and the idea of splitting a number into its parts (place value) to help understanding of the process.

How many sweets do I need for 24 party bags if each is to have 6 sweets?

$$\begin{array}{r} \times \quad 20 \quad 4 \\ 6 \quad \boxed{120} \quad \boxed{24} \quad = 144 \text{ sweets} \end{array}$$

You will see the 24 has been split into 20 and 4, each has been multiplied by 6 mentally and the two numbers added to give the final total. Many children will eventually develop the ability to do this kind of calculation totally in their heads.

Here is a slightly more difficult example.

How much does it cost if I buy 9 books at 72p each?

$$\begin{array}{r} \times \quad 70 \quad 2 \\ 9 \quad \boxed{630} \quad \boxed{18} \quad = 648 \end{array}$$

It costs 648p or £6.48

The expanded method for division is often called 'chunking' and really just involves partitioning the number into helpful 'chunks' related to the number you are dividing by (divisor) or counting on/taking away chunks of the same size until you run out. It uses the fact that division is repeated subtraction of the same size group. So $20 \div 4 = 5$ involves subtracting 4s from 20 until it's been used up or counting on in fours until you reach 20. You can do this 5 times.

Here is a more difficult example showing how larger chunks are used to speed up the process. Again this method uses and builds on the ideas explained earlier.

72 pears are packed in boxes of 6. How many boxes would there be?
 First partition the 72 into chunks related to multiples of the divisor
 $72 \div 5$ lies between $50 \div 5 = 10$ and $100 \div 5 = 20$

72	
- 50	(10 groups) or (10 x 5)
22	
- 20	(4 groups) or (4 x 5)
2	

Answer : 14 remainder 2

As they move into upper Key Stage 2 children will still be encouraged to choose the most suitable method of calculation, mentally if possible. Where this is not possible they will be using expanded or compact methods and a calculator for more complex and involved work.

Most children will be expected to multiply 3 digits x 1 digit and 3 digits x 2 digits using a written method. Here are some examples.

How many hours are there in the year 2003?
 This means we have to do 365×24 ...a calculation you may find quite hard. Here it is expanded and using lots of mental skills...but none of them difficult.

x	300	60	5	
20	6000	1200	100	= 7300
4	1200	240	20	= 1460

So $20 \times 365 = 7300$
 $4 \times 365 = 1460$
 giving a total of 8760 hours with no difficult calculation to do.

More complex division will involve dividing 3 digit numbers by a 1 digit number and 3 digit numbers by a 2 digit number. With division, as with all calculation, it's important to think about what the actual problem is asking when you come to

give an answer. This is shown up in the second example of expanded division or chunking involving buses for a school trip.

458 stickers are shared between 3 children. How many does each get?

4 5 8	
-3 0 0	that's 100 x3
1 5 8	left
-1 5 0	that's 50 x3
8	left
-6	that's 2 x3
2	left over

So each gets **100+50+2** or 152 stickers with 2 left over.
 $458 \div 3 = 152$ remainder 2

432 children and adults are going on a school trip. If each bus takes 30 people how many are needed?

4 3 2	people going	
-3 0 0	that's 10 x30	or 10 buses
1 3 2	left	
-1 2 0	that's 4 x30	or 4 buses
1 2	left	1 bus

So we see that the calculation would result in
 $432 \div 30 = 14$ remainder 12

This is not a good answer for this question because the 12 people left over would need another bus or they couldn't go!

So we see that 15 buses are needed...or some cars.

When the children really understand these expanded methods they can be shown how they are developed into a compact method. Remember though that the expanded methods are perfectly good ways of working out an answer if the children feel more comfortable and therefore find it easier. They give the same answer and it can often be quicker if they are confident about what they are doing.

Pupils then use more formal methods for short multiplication, beginning with the expanded version:

$$\begin{array}{r} 87 \\ \times 6 \\ \hline 42 \quad (7 \times 6) \\ 480 \quad (80 \times 6) \\ \hline 522 \quad (\text{ones, then tens, hundreds etc}) \end{array}$$

Leading onto the shorter version:

$$\begin{array}{r} 87 \\ \times 6 \\ \hline 522 \\ \hline 4 \end{array}$$

Long multiplication:

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 42 \quad (6 \times 7) \\ 350 \quad (50 \times 7) \\ 120 \quad (6 \times 20) \\ 1000 \quad (50 \times 20) \\ \hline 1512 \\ 1 \end{array}$$

followed by

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 392 \quad (56 \times 7) \\ 1120 \quad (56 \times 20) \\ \hline 1512 \end{array}$$

Hopefully reading this has made you feel a little more confident and comfortable with numbers and calculations yourself, if you weren't already, and therefore better able to help your child. Remember if you have any concerns or questions it's always better to talk to your child's teacher rather than worrying about what's going on. Children (and adults) need to feel confident with numbers and to enjoy playing with them and using them, that's really what it's all about. It then means using them for everyday purposes becomes a doddle rather than a threat.